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Improved Method for the Initial Attitude Acquisition Maneuver

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Introduction

FOR the initial attitude acquisition maneuver of dual-spin satellites, whose initial spin axis is not that of the maximum moment of inertia, Hubert[1] developed a counterpart to the dual-spin turn of Kaplan and Patterson.[2] With a passive energy damper to dissipate core energy, the technique uses the momentum wheel to remove the ambiguity in the polarity of the final attitude. In the final attitude, let us call "right-side-up" the case where the wheel's angular momentum vector, as seen from the body frame, is aligned with the inertial one and we will call the opposite case "upside-down." In Hubert's technique, the wheel is maintained so that its angular momentum is always above a certain percentage of the total momentum. This raises the energy level of the upside-down attitude while also destabilizing it. Thus, after all the core energy is dissipated, the final attitude will be the right-side-up one. In this paper, a new technique, along the same lines as Hubert's but without the passive energy damper, is presented. For this new technique, the wheel itself is used as an active core energy dissipator while simultaneously maintaining the conditions necessary to guarantee the final attitude. This eliminates the need for the passive energy damper, saving both weight and expense. The only measurements required for the maneuver are the polarity of a component of the body's angular velocity and the wheel's speed. Both of these can usually be provided by instruments normally onboard.

Analysis

The maneuver is developed for a rigid satellite initially spinning about its axis of minimum moment of inertia. The purpose of the maneuver is to transfer the momentum to a rigid symmetric momentum wheel aligned with the axis of maximum moment of inertia. The principal moments of inertia I_1, I_2, and I_3 are defined to include the apppropriate components of the wheel inertia I_w, and satisfy the condition

$$I_3 > I_2 > I_1 \tag{1}$$

In the absence of external torques, the motion is described by the equations

$$\begin{aligned}\dot{\omega}_1 &= -\omega_2[(I_3-I_2)\omega_3 + I_w\Omega]/I_1\\ \dot{\omega}_2 &= \omega_1[(I_3-I_1)\omega_3 + I_w\Omega]/I_2 \\ \dot{\omega}_3 &= -[\omega_1\omega_2(I_2-I_1)+T]/(I_3-I_w)\end{aligned} \tag{2}$$

$$\dot{\Omega} = T/I_w - \dot{\omega}_3 \tag{3}$$

where ω are the angular velocities, as seen from the body frame, around the respective axes of inertia, and Ω is the angular velocity of the wheel relative to the body. T is the command torque, and friction is neglected here. The core energy E_c is defined as the total energy excluding that part of the energy due solely to the relative motion of the wheel[3]

$$E_c = ½(I_1\omega_1^2 + I_2\omega_2^2 + I_3\omega_3^2) \tag{4}$$

The time rate of change for E_c is given by

$$\dot{E}_c = I_1\omega_1\dot{\omega}_1 + I_2\omega_2\dot{\omega}_2 + I_3\omega_3\dot{\omega}_3 \tag{5}$$

Substituting Eqs. (2) and (3) into Eq. (5) yields

$$\dot{E}_c = \frac{-\omega_3}{I_3-I_w}[\omega_1\omega_2 I_w(I_2-I_1) + I_3 T] \tag{6}$$

In order to use the wheel successfully as an energy dissipator, we must guarantee that $\dot{E}_c$ is negative. There are many ways to control the wheel so that this is true. Perhaps the simplest and the easiest to implement is a switched control law of the form

$$T = T_0\,\mathrm{sign}(\omega_3) \tag{7}$$

where T_0 is a positive constant and satisfies the following,

$$T_0 > \frac{I_w}{I_3}\,\frac{h_0^2(I_2-I_1)}{2I_1I_2} \tag{8}$$

and h_0 is the total momentum. We now define the normalized wheel momentum:

$$\Phi = \frac{I_w\Omega}{h_0} \tag{9}$$

The threshold Ψ, defined by Hubert, is

$$\Psi = \frac{I_3-I_2}{I_2} \tag{10}$$

Provided the condition

$$\Phi \geq \Psi \tag{11}$$

is satisfied during separatrix crossing, then the separatrix leading to the right-side-up attitude will be crossed and the correct final attitude will result. Here we will try to use this same threshold technique to guarantee the final attitude while using the wheel itself as an active energy damper, by implementing a combination of Eqs. (7) and (11).

As a guide to understanding the motion, let us construct the dual-spin polhode curves. If we define h_3^* as the total momentum along the 3 axis,

$$h_3^* = I_3\omega_3 + I_w\Omega \tag{12}$$

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then we can construct the familiar momentum sphere defined by

$$h_0^2 = h_1^2 + h_2^2 + h_3^{*2}$$

$$h_0 = (I_1\omega_1)^2 + (I_2\omega_2)^2 + (I_3\omega_3 + I_w\Omega)^2 \tag{13}$$

and the core energy ellipsoid

$$2E_c = \frac{h_1^2}{I_1} + \frac{h_2^2}{I_2} + \frac{(h_3^* - I_w\Omega)^2}{I_3} \tag{14}$$

The most helpful form of polhode drawing is the projection onto the 1,3 plane of the separatrices and nearby curves. The separatrices are a pair of intersecting straight lines centered at

$$h_1 = 0 \qquad h_3^* = -I_w\Omega\frac{I_2}{I_3 - I_2} \tag{15}$$

with slope m

$$m = \pm\sqrt{\frac{I_3}{I_1}\,\frac{I_2 - I_1}{I_3 - I_2}} \tag{16}$$

A sketch of the polhodes for various ranges of Φ appears in Fig. 1. It is interesting here to note that when Eq. (11) is satisfied corresponding to Fig. 1b or 1c, there are only three sections on the polhode. The two outer portions correspond to high-energy states, so only one is left for low-energy states. This low-energy section also happens to contain the desired final altitude. Thus, when the polhode has only three sections, complete core energy dissipation will result in the correct final attitude as predicted by Hubert.[1]

If we now restrict ourselves temporarily to the case where Eq. (11) is satisfied, four possible states emerge. They are as follows:

$$\Phi > \Psi \qquad \omega_3 > 0 \tag{17a}$$

$$\Phi > \Psi \qquad \omega_3 < 0 \tag{17b}$$

$$\Phi = \Psi \qquad \omega_3 > 0 \tag{17c}$$

$$\Phi = \Psi \qquad \omega_3 < 0 \tag{17d}$$

In order to determine the action to be taken for each of these cases, let us once again consider the motor torque equation (3). Since $I_3 \gg I_w$, then in general $\dot\omega_3 \ll \dot\Omega$. For the following analysis, let us assume that T and $\dot\Omega$ and therefore $\dot\Phi$ will have the same sign. Now let us consider the effect of the torques suggested by Eq. (7) on the states of Eqs. (17).

For the state of Eq. (17a), Eq. (7) suggests a positive torque, which will yield a positive $\dot\Phi$. This will not violate Eq. (11) so it should be implemented. For the case of Eq. (17b), Eq. (7) suggests a negative torque, which will produce a negative $\dot\Phi$. Since $\Phi > \Psi$ by definition of Eq. (17b), this will not violate Eq. (11), so the torque of Eq. (7) can be implemented. For the case of Eq. (17c), Eq. (7) suggests a positive torque, which will produce a positive Φ, thus not driving Φ below Ψ, so Eq. (7) can be implemented. For the case of Eq. (17d), however, the negative torque and thus negative $\dot\Phi$ suggested by Eq. (7) would result in a violation of Eq. (11). So in this case, Eq. (7) cannot be used. Rather, it would be better to place the satellite in a "holding" position and hope that the polhode will naturally revert to one of the other states, whereupon Eq. (7) can be resumed again. We must thus first design an adequate "holding" pattern and then define the conditions under which Eq. (17d) will always naturally revert to another state of Eqs. (17).

As far as a "holding" state is concerned, we must make sure that it will not violate Eq. (11), and we would also like one that does not increase the core energy, as this would be counter-

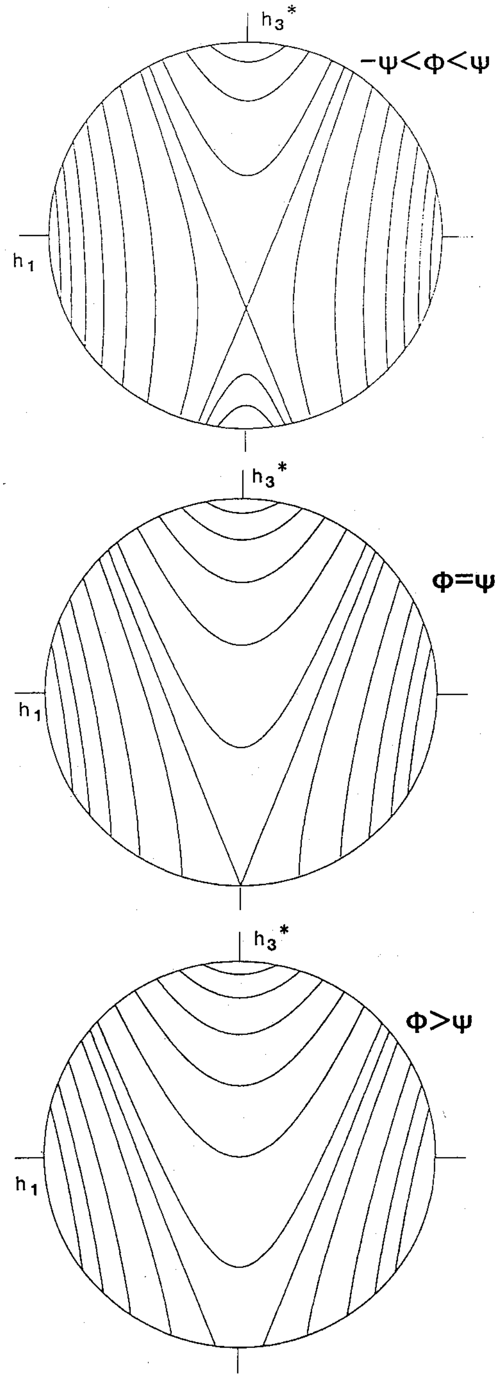

Fig. 1 Polhode for various values of normalized wheel momentum.

productive. A control law that satisfies both these criteria is to command the motor to maintain Φ constant. This law is fairly simple to implement in hardware, as the wheel tachometer can be used to measure Ω directly.

In order for Eq. (17d) to revert naturally to anothe state, either Φ must grow or ω_3 must change sign. By definition of our holding pattern $\dot\Phi = 0$, so we must therefore find the conditions that guarantee that ω_3 will once again become positive.

Let us again look at Fig. 1b with the dashed line $h_3^* = h_0\Phi + h_0\ \Psi$ drawn in, as shown in Fig. 2. In the region above the dashed line, $h_3^* > I_w\Omega$ or alternatively $\omega_3 > 0$. If the polhode is divided into three zones, two lying to the outside of the separatrices and one on the inside, it can be seen that, depending on the slope of the separatrices, there are at least some paths in each zone for which a portion of that path lies above the dashed line. The limiting value of this slope is drawn in Fig. 2 and given by

$$s = \pm\sqrt{I_3/(2I_2 - I_3)} \tag{18}$$

Provided that m of Eq. (14) is greater than s of Eq. (18), then at least some of the paths in the outer zones as well as all of the paths in the inner zone will have portions above the dashed line. Along these paths, the case of Eq. (17d) will naturally revert to that of Eq. (17c). The condition for $m > s$ is as follows:

$$I_2 > (I_3 + I_1)/2 \tag{19}$$

This is easily interpreted by saying that the intermediate moment of inertia I_2 must be closer to the maximum I_3 than it is to the minimum I_1.

Even when Eq. (19) is satisfied, however, it is quite apparent that not all the paths in the outer zones will cross over the dotted line. Those paths that do not cross the dotted line correspond to higher energy levels compared to those that do. For these high-energy level states, Eq. (17d) will not revert naturally to another state of Eq. (17), so it is futile to place the satellite into the "holding" pattern previously devised. Any attempt to use Eq. (7) while satisfying Eq. (11) is also futile. The limiting energy level is given by

$$E_c \leq [h_0^2 - (I_w\Omega)^2]/2I_1 \tag{20}$$

It is thus obvious that before Eq. (7) can be used while Eq. (11) is simultaneously satisfied, energy must first be dissipated below the level of Eq. (20), whereupon Eqs. (7) and (11) can then be used in tandem.

This initial energy dissipation can be accomplished in many ways. Again, perhaps the simplest of these is to use the control law of Eq. (7) without regard as to whether Eq. (11) is satisfied or not. For the entire maneuver, our analysis suggests using the following sequence:

Use Eq. (7) until a separatrix is crossed (21a)

Spin up wheel to satisfy Eq. (11) (21b)

Use Eq. (7) while also satisfying Eq. (11) (21c)

It should be noted here that when Eq. (19) is satisfied, the energy level of the separatrix will be lower than that of Eq. (20). Since during Eq. (21a) Eq. (11) is not satisfied, the separatrices will be of the form of Fig. 1a. When the law of

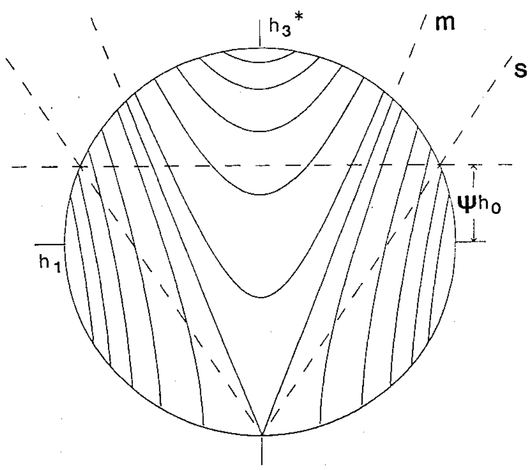

Fig. 2 Polhode for wheel momentum equal to Hubert threshold level.

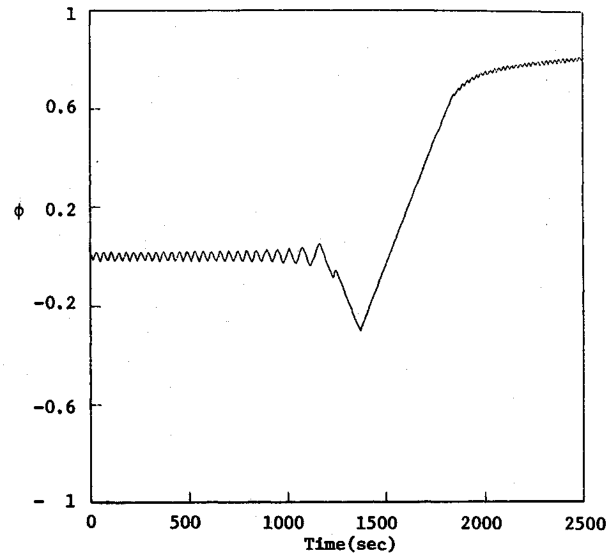

Fig. 3a Normalized wheel momentum vs time.

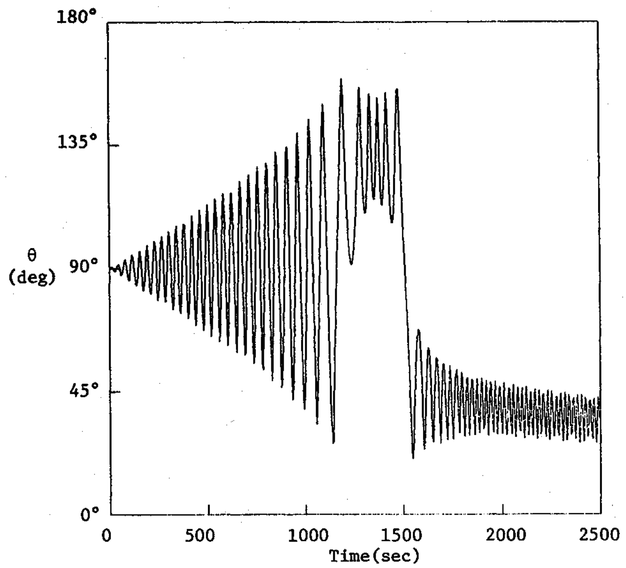

Fig. 3b Nutation angle vs time.

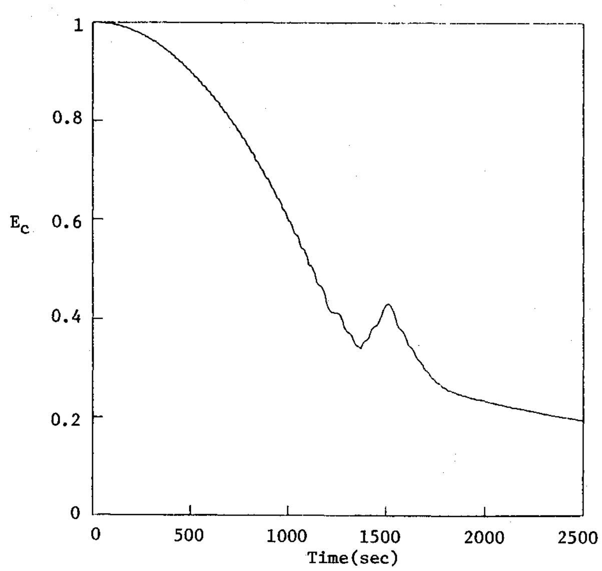

Fig. 3c Normalized core energy vs time.

Eq. (7) alone is used, it is entirely possible that after the separatrix crossing the satellite might initially find itself on the bottom portion of the polhode. The implementation of Eq. (21b) will destabilize this upside-down portion of the polhode, however, so it is not really bothersome that this might initially occur.

For hardware implementation, all that is needed for Eqs. (21) is the motor tachometer and a sensor to detect the polarity of ω_3 [for the implementation of Eq. (7)]. The separatrix can be detected by keeping track of the zero crossings of Φ. Soon after traversing a separatrix, Φ will approach either $+1$ or -1. In any case, it will cease crossing zero regularly so that when zero crossings stop occurring, we can say that a separatrix has been traversed.

A cautionary note concerns the spin-up stage of Eq. (21b): It is likely that the spin-up of the wheel will result in an increase in core energy. Care must be taken then to assure that this increase does not push E_c to the point where it violates Eq. (20). Assuming that the sequence outlined in Eqs. (21) is used and provided that the gap between s of Eq. (18) and m of Eq. (14) is large, this should not present a problem. If s and m are quite close to each other, care must be taken during spin-up, probably by implementing a complex spin-up law, to minimize any possible increase in E_c.

Simulations

Simulations were performed to assess the performance of the proposed technique. The parameters used are those of a small scientific satellite that carries a long thin telescope. The principal moments of inertia are $I_1 = 70$ kg m^2, $I_2 = 170$ kg m^2, $I_3 = 190$ kg m^2, and $I_w = 0.06$ kg m^2. The value of T_0 is 0.04 Nm. The simulations were carried out using the sequence of Eqs. (21). The initial conditions are spin about the 1 axis at 3 rpm with a very small nutation angle. The maneuver is considered over when the nutation angle reaches below 10 deg.

Plots of the wheel momentum, or energy, and nutation angle appear in Fig. 3. The preceding case was chosen specifically to show the result when the separatrix naturally crossed as a result of Eq. (21a) is the upside-down one. That case serves to illustrate the way in which the wheel is spun up so as to destabilize the upside-down attitude. A separatrix was first crossed at $T = 1250$ s. From the plot of the nutation angle between $T = 1250$ s and $T = 1500$ s, it can be seen that this separatrix was the upside-down one and would have resulted in a final nutation angle of 180 deg. At $T = 1370$ s, the onboard controller, by monitoring zero crossings of the wheel speed, determined that a separatrix has been crossed, so Eq. (21b) was implemented and the wheel spun up. The core energy rise associated with the spin-up is easily seen. After the spin-up Eq. (21c) was implemented, and the nutation angle gradually approached 10 deg, the desired final attitude. The total time required for the maneuver was 4.5 h. The total energy consumed by the motor was 6.9% of the onboard battery's capacity.

Conclusion

An improved technique for the initial attitude acquisition maneuver of a class of satellites has been presented. The new technique dispenses with the need for a special passive energy damper, thus saving cost and weights. Its implementation requires that one relation of the moments of inertia be satisfied, but otherwise can probably be implemented using equipment normally onboard.

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